Test: Binomial Situations and Sampling Distributions
Instructor's Version

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Answers to the Free Response Questions: 3 questions, 52 points, 30 minutes

1. Annie is a basketball player who makes, on average, 65% of her free throws. Assume each shot is independent and the probability of making any given shot is .65.

A. What's the probability Annie will miss three straight free throws before she makes one? (If you use a calculator to get your answer, write your answer in standard notation, not calculator notation.) (5 points)

Answer
Calculate the probability of getting a success on the fourth try in a geometric distribution with a probability of success of .65 on each trial:
G(4) = (.65)(.35)^3 = .028.

NOTE to instructor: on a TI-83, this can be solved by geometpdf(.65,4), or (1/4)binompdf(4,.65,1).

B. During a season, Annie takes 140 free throws. What's the exact binomial probability she'll make at least 100 out of 140 of these throws? (4 points)

Answer

$$\binom{140}{100}(.65)^{100}(.35)^{40} + \binom{140}{101}(.65)^{101}(.35)^{39} + \ldots + \binom{140}{140}(.65)^{140}(.35)^{0}$$

NOTE to the instructor: On the TI-83 this is 1–binomcdf(140,.65,99) = .064.

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C. For part B, are the conditions that permit you to use a normal approximation to the binomial satisfied? Explain. (5 points)

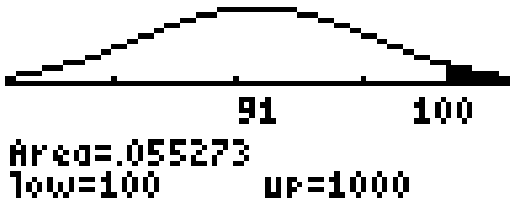
Answer
Yes; 140(.65) = 91 ≥ 10, 140(.35) = 49 ≥ 10.

(NOTE: It's also okay if the student writes 140(.65) = 91 ≥ 5, 140(.35) = 49 ≥ 5.)

D. Redo part B using a normal approximation, <u>without</u> continuity correction, to the binomial. Draw a sketch of the situation; draw the distribution and shade the area representing the probability you're finding. (5 points)

NOTE: If you use the TI-83, you don't need to calculate the *z*-scores. However, you should explain your answer in enough detail to show your instructor you understand what you're doing.

Answer
$\mu_X = .65(140) = 91,\ \ \sigma_X = \sqrt{140(.65(.35)} = 5.64$



z = (100 − 91)/5.64 = 1.60 ➔ A = 1 − 0.945 = 0.055

NOTES to the instructor: If they get the right answer, give full credit if they don't show the *z*-scores. On a TI-83 this is normalcdf(100,1000,91,5.64) = .055.

If students use the TI-83 they don't need to calculate *z*-scores, but they should explain their answers in terms that show they know what they're doing.

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E. Redo part B using a normal approximation, <u>with</u> continuity correction, to the binomial. NOTE: If you use the TI-83 you don't need to calculate *z*-scores, but do explain your answer in enough detail to show your instructor you understand what you're doing. (5 points)

Answer
$z = (99.5 - 91)/5.64 = 1.507$ ➔ $A = 1 - .934 = .066$

NOTE to the instructor: On a TI-83 this is normalcdf(99.5,1000,91,5.64)=.066. If students use the TI-83 they don't need to calculate *z*-scores, but they should explain their answers in terms that show they know what they're doing.

2. A magazine reports that the average bowling score for league bowlers in the United States is 157 with a standard deviation of 12, and that the scores are approximately normally distributed.

A. State the shape, the mean, and the standard deviation of the mean $\bar{x}$ of 15 bowlers selected at random. Justify each statement by showing your work or identifying the theorem that tells you your answer is correct. (4 points)

Answer
Since the parent population is approximately normal, we expect the sampling distribution also to be approximately normal. The mean of the sampling distribution is 157 and the standard deviation is 12/sqrt(15) = 3.098.

B. What's the probability that a random sample of 15 bowlers would have an average score greater than 165? (5 points)

Answer
$z = (165 - 157)/3.098 = 2.58.$
$P(z > 2.58) = .005$

NOTE to instructor: On the TI-83 this is normalcdf(165,1000,157,3.098) = .005. If students use the TI-83 they don't need to calculate *z*-scores, but they should explain their answers in terms that show they know what they're doing.

C. What's the probability a random sample of 150 bowlers will have an average score greater than 165? (5 points)

Answer
$z = (165 - 157)/.9798 = 8.165$
$P(z > 8.165) = 1.63 \times 10^{-16}$

NOTE to instructor: On the TI-83 this is normalcdf(165,1000,157,12/sqrt(150) = 1.63 x 10^(−16). If students use the TI-83 they don't need to calculate *z*-scores, but they should explain their answers in terms that show they know what they're doing.

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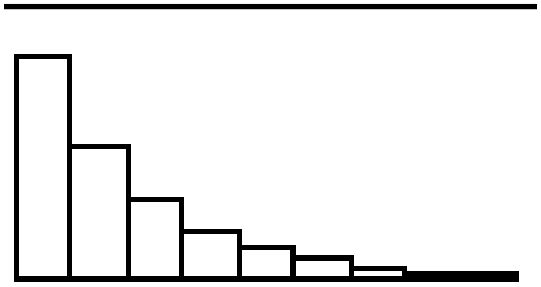
3. The following is a histogram of the first ten terms of a geometric distribution with $p = .4$.



The mean of this distribution is 2.5 and the standard deviation is approximately 1.93.

A. What's the shape of the distribution of sample means for simple random samples of size 5 drawn from this distribution? What are the mean and the standard deviation of the sampling distribution? (5 points)

Answer
For samples of size 5, the sampling distribution would be somewhat skewed to the right, like the parent population. Its mean would be 2.5 and its standard deviation would be 1.93/sqrt(5) = .86.

B. What's the shape of the distribution of sample means for simple random samples of size 50 drawn from this distribution? (5 points)

Answer
The Central Limit Theorem tells us we'd expect the distribution of x-bar for samples of size 50 to be approximately normal and centered at 2.50.

C. Suppose we know this event didn't succeed on the first 5 trials. On average, how long will we have to wait for the first success to occur? (4 points)

Answer
We'd have to wait 1/.4 = 2.5 trials.

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